Fundamental Trigonometric Identities

What is an **identity**?

1. $x^3 - a^3 = (x-a)(x^2 + ax + a^2)$

   is true for all real values of x and a. It is an identity.

2. $x^2 - 3x + 2 = 0$ is not an identity since it is only true for $x = 1$ and $x = 2$.

   $$\left(\begin{array}{l} x^3 - 3x + 2 = (x-1)(x-2) = 0 \\ x = 1 \text{ or } x = 2 \end{array} \right)$$

3. $\sin^2\theta + \cos^2\theta = 1$ is true for all values of θ (radians), and is a trigonometric identity.

Fundamental Identities:

Reciprocal Identities:

$sin\,\theta = \frac{1}{csc\,\theta}$ $\qquad$ $cos\,\theta = \frac{1}{sec\,\theta}$ $\qquad$ $tan\,\theta = \frac{1}{cot\,\theta}$

$csc\,\theta = \frac{1}{sin\,\theta}$ $\qquad$ $sec\,\theta = \frac{1}{cos\,\theta}$ $\qquad$ $cot\,\theta = \frac{1}{tan\,\theta}$

Quotient Identities:

$tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$ $\qquad$ $cot\,u = \frac{cos\,\theta}{sin\,\theta}$

Pythagorean Identities:

$sin^2\,\theta + cos^2\,\theta = 1$; $cos^2\,\theta = 1 - sin^2\,\theta$; $sin^2\,\theta = 1 - cos^2\,\theta$

$1 + tan^2\,\theta = sec^2\,\theta$ $\qquad$ $1 + cot^2\,\theta = csc^2\,\theta$

Cofunction Identities:

$sin\left(\frac{\pi}{2} - \theta\right) = cos\,\theta$ $\qquad$ $tan\left(\frac{\pi}{2} - \theta\right) = cot\,\theta$ $\qquad$ $sec\left(\frac{\pi}{2} - \theta\right) = csc\,\theta$

$cos\left(\frac{\pi}{2} - \theta\right) = sin\,\theta$ $\qquad$ $cot\left(\frac{\pi}{2} - \theta\right) = tan\,\theta$ $\qquad$ $csc\left(\frac{\pi}{2} - \theta\right) = sec\,\theta$

Even/Odd Identities:

$sin(-\theta) = -sin\,\theta$ $\qquad$ $cos(-\theta) = cos\,\theta$ $\qquad$ $tan(-\theta) = -tan\,\theta$

$csc(-\theta) = -csc\,\theta$ $\qquad$ $sec(-\theta) = sec\,\theta$ $\qquad$ $cot(-\theta) = -cot\,\theta$

Simplify the Expressions Below:

$$\frac{\sec\theta}{\csc\theta} = \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$(\cos\theta \neq 0, \sin\theta \neq 0)$

$$\tan x \cos(-x) = \frac{\sin x}{\cos x} \cdot \cos x = \sin x$$

$(\text{for } \cos\theta \neq 0,\ \theta \neq \frac{\pi}{2} \pm k\pi)$

$$\cos\theta \sec\theta - \cos^2\theta = \cos\theta \cdot \frac{1}{\cos\theta} - \cos^2\theta$$

$$= 1 - \cos^2\theta = \sin^2\theta.$$

$(\text{for } \cos\theta \neq 0,\ \theta \neq \frac{\pi}{2} \pm k\pi)$

Verify the Identity: $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

$$\frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)} = \frac{2}{\sin\theta}$$

$$\frac{1 + 1 + 2\cos\theta}{1+\cos\theta} = 2$$

$$\frac{2+2\cos\theta}{1+\cos\theta} = \frac{2(1+\cos\theta)}{(1+\cos\theta)} = 2$$

$(\sin\theta \neq 0,\ \cos\theta \neq -1)$

Verify the Identity: $\sin\theta + \sin(-\theta)\cos^2\theta = \sin^3\theta$

$$\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$$

$$\sin\theta(1-\cos^2\theta) = \sin^3\theta$$

$$\sin\theta(\sin^2\theta) = \sin^3\theta$$

Verify the Identity: $\frac{sin^4 x - cos^4 x}{sin^2 x - cos^2 x} = 1$

$$\frac{\cancel{(sin^2x - cos^2x)}(sin^2x + cos^2x)}{\cancel{(sin^2x - cos^2x)}}$$

$= sin^2x + cos^2x = 1$

Verify the Identity

$sin\, x\,(sin\, x - cos\, x) + cos\, x\,(sin\, x + cos\, x) = 1$

$sin^2x - sin\,x\,cos\,x + cos\,x\,sin\,x + cos^2x$

$= sin^2x + cos^2x = 1$

Verify the Identity

$$\underbrace{-\cos^7 x\ \sin x - \cos^5 x\ \sin x}_{\text{factor out } \cos^5 x \sin x} = \cos^5 x\ \sin^3 x$$

$$\cos^5 x\ \sin x\left[-\cos^2 x + 1\right]$$

$$= \cos^5 x \cdot \sin x\left[\sin^2 x\right]$$

$$= \cos^5 x\ \sin^3 x$$

Guidelines for Verifying an Identity

1. Work with one side of an equation at a time. It is often better to work with the more complicated side.

2. Look for opportunities to factor an expression, add fractions, multiply and expand an expression.

3. Look for opportunities to use fundamental identities.

4. If all else fails, try converting all trigonometric expressions to be in terms of sine and cosine.

5. Try something, be motivated and not discouraged.